PHYSICAL REVIEW E VOLUME 61, NUMBER 1 JANUARY 2000

Nonlinear dust kinetic Alfvén waves

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(Received 3 August 1999)

Localized nonlinear dust kinetic Alfvén waves are investigated. It is found that finite density dips and humps can coexist. The density humps are cusped and narrower than the dips.

PACS number(s): 52.35.Sb, 52.35.Mw, 52.25.Wz, 94.20.Vv

I. INTRODUCTION

Kinetic Alfvén waves (KAWs) propagating obliquely to an external magnetic field in plasmas have been investigated extensively because of their relevance to the interpretation and diagnostics of electromagnetic fluctuations in space and laboratory plasmas. Data from the Freja spacecraft showed that events of spiky density pulses exhibiting KAW field characteristics occur in the auroral region of the Earth's ionosphere [1,2]. Hasegawa and Mima [3] and others [4–7] investigated nonlinear KAWs in a low-β ($m_e/m_i \ll \beta \ll 1$, where $\beta \equiv 4\pi n_i T/B_0^2$, m_e and m_i are the electron and ion masses, n_i is the ion density [8], and T is the effective temperature) collisionless plasma. It was found that nonlinear localized wave structures can exist.

Heavy, highly charged dust grains often appear in ionospheric and other plasmas. Such massive charged dust grains not only change the properties of the existing plasma, but can also introduce new wave modes [9–12]. Several authors have studied coherent structures in an unmagnetized dusty plasma [10,13–16]. It was found that very low frequency fluctuations on the very long dust timescale can be excited. In this paper, we investigate nonlinear dust KAWs (DKAWs), which are Alfvén-like waves driven by the polarization drift of the dusts and bending of the magnetic field lines. It is found that solitons involving smooth density dips as well as cusped density humps can exist. The results, especially that on the cusped soliton because of its unique properties, may be useful for the diagnostics of dusts in magnetized plasmas.

For very low-frequency perturbations on the dust timescale, the electrons and ions are fully relaxed and in local thermodynamic equilibrium, obeying Boltzmann density distributions. The dusts are in general cold and their motion strongly affected by the external magnetic field. The response of the dust fluid (via its polarization drift) to the transverse magnetic field line perturbation can then lead to the appearance of DKAWs. Furthermore, the electron and ion pressures acting on the dust grains via the self-consistent electrostatic field can lead to effective, or averaged, dust Larmor motion. The latter causes dispersion of the DKAWs in the direction perpendicular to the external field. A balance of the wave dispersion and nonlinearity can give rise to localized quasistationary wave structures.

II. FORMULATION

We consider a collisionless plasma consisting of electrons, ions, and massive highly charged dust grains. The constant external magnetic field is given by $\vec{B}_0 = B_0 \hat{e}_z$, where $\hat{e}_z$ is the unit vector along the z direction. Since the electrons are much faster than the ions, the charge on the dust grain is usually negative.

We assume $v_{Ad} \gg v_{sd}$, where $v_{sd} = (Z_d T/m_d)^{1/2}$ and $v_{Ad} = (B_0^2/4\pi n_{d0} m_d)^{1/2}$ are the dust acoustic and dust Alfvén speeds, and n_{d0}, m_d, and $-Z_d =$ const are the equilibrium dust density, dust mass, and average dust charge, respectively. In terms of β the above condition is $\beta \ll n_{i0}/n_{d0} Z_d$, or $\beta_d \equiv 4\pi n_{d0} T/B_0^2 \ll 1/Z_d$. Note that $Z_d \beta_d$ can be of the same order as β. One can therefore neglect the compressional component of the wave magnetic field and avoid the linear coupling between the DKAWs to the dust acoustic waves. We also assume $v_{ti} \gg v_{Ad}$ (i.e., $\beta_d \gg m_i/m_d$), $\max\{\nu_{di}, \nu_{de}, \nu_{dd}\} \ll k v_{Ad}$, where v_{ti} is the ion thermal speed, ν_{dj} $(j = e,i,d)$ are the dust collision frequencies, and $L^{-1} v_{Ad}$ is the characteristic transit time of the DKAW structure of size L. That is, dust-electron, dust-ion, and dust-dust collisions, which are all much less than that of the electron and ion collisions, are neglected. Thus, for the DKAW motion the electrons and ions remain in thermal equilibrium, the dust fluid remains cold, the average dust charge remains constant, and the electron and ion skin-depth effects can be ignored. The latter can modify the wave dispersion.

As the KAWs contain a transverse magnetic component, it is convenient to introduce [3] for the electric field the longitudinal component $E_x = -\partial_x \phi$ and a mixed component $E_z = -\partial_z \psi (= -\partial_z \phi - c^{-1} \partial_t A_z$, where A_z is the vector potential parallel to the external field). The Maxwell equations lead to

$$\partial_t B_y = c\, \partial_x \partial_z (\phi - \psi), \tag{1}$$

$$\partial_x^2 \partial_z^2 (\phi - \psi) = (4\pi/c^2)\, \partial_t \partial_z j_z, \tag{2}$$

where j_z is the longitudinal current. The densities of the thermal electrons and ions are given by

$$n_e = n_{e0} \exp(e\psi/T_e), \tag{3}$$

$$n_i = n_{i0} \exp(-e\psi/T_i), \tag{4}$$

where T_e and T_i are the electron and ion temperatures, respectively.

The dust are cold and strongly magnetized. In the drift approximation ($\partial_t \ll \Omega_d$, where $\Omega_d = Z_d e B_0 / m_d c$ is the dust cyclotron frequency), the dust continuity equation can be written as

$$\partial_t n_d + \frac{c}{B_0 \Omega_d} \partial_x (n_d \partial_x \partial_t \phi) = 0, \tag{5}$$

where we have neglected v_{dz}, which would contribute to wave dispersion of higher order in β_d and lead to a linear coupling with the dust acoustic waves. The system of equations (1)–(5) is closed by the charge neutrality and the current continuity equations

$$n_e + Z_d n_d - n_i = 0, \tag{6}$$

$$\partial_z j_z = e \partial_t (n_e - n_i), \tag{7}$$

where we have noted that $\nabla \cdot j_\perp = -\partial_z j_z$, $j_\perp \approx j_{d\perp}$, and $j_z \approx j_{ez} + j_{iz}$. Equations (1)–(7) govern nonlinear dust KAW motion in a dusty plasma. Terms which are both nonlinear and of order higher than $\partial_t / \Omega_d (\ll 1)$ have been neglected.

III. DUST KINETIC ALFVÉN WAVES

Normalizing Eqs.(1)–(7), we obtain

$$N_e = \exp(\Psi), \tag{8}$$

$$N_i = \exp(-\sigma \Psi), \tag{9}$$

$$\partial_\tau N_d + \partial_\xi (N_d \partial_\xi \partial_\zeta \Phi) = 0, \tag{10}$$

$$\partial_\xi^2 \partial_\zeta^2 (\Phi - \Psi) = \frac{1}{1-\delta_e} \partial_\tau^2 (\delta_e N_e - N_i), \tag{11}$$

$$\delta_e N_e + Z_d \delta_d N_d - N_i = 0, \tag{12}$$

where $\tau = \Omega_d t$, $\xi = x/\rho_{sd}$, $\zeta = z \omega_{pd}/c$, $(\Phi, \Psi) = e(\phi, \psi)/T_e$, $N_j = n_j / n_{j0}$ $(j = e, i, d)$, $\delta_e = n_{e0}/n_{i0}$, $\delta_d = n_{d0}/n_{i0}$, $\sigma = T_e / T_i$, $\omega_{pd} = (4\pi n_{d0} Z_d^2 e^2 / m_d)^{1/2}$ is the dust plasma frequency, and $\rho_{sd} = (Z_d T_e / m_d \Omega_d^2)^{1/2}$ is the dust gyroradius. Note that $\delta_d = (1-\delta_e)/Z_d$.

Linearizing Eqs. (8)–(12) and assuming that the perturbed variables are of the form $\exp i(k_\perp \xi + k_\parallel \zeta - \omega \tau)$, we obtain the (dimensionless) linear dispersion relation

$$\omega^2 = k_\parallel^2 \left(1 + k_\perp^2 \frac{1-\delta_e}{\delta_e + \sigma} \right), \tag{13}$$

for the DKAWs. In the dimensional form it is

$$\omega^2 = k_\parallel^2 v_{Ad}^2 \left(1 + k_\perp^2 \rho_{sd}^2 \frac{n_{i0} - n_{e0}}{n_{e0} + n_{i0} T_e / T_i} \right), \tag{14}$$

so that the finite dust Larmor radius ρ_{sd} leads to dispersion of the DKAWs. We see that the wave dispersion increases with the dust density.

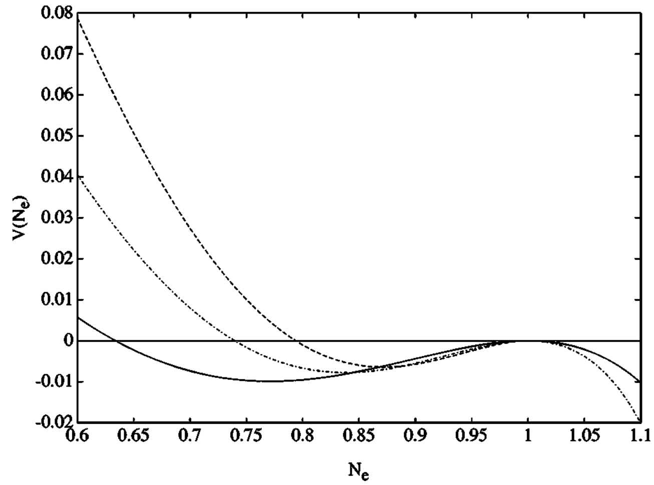

FIG. 1. The Sagdeev potential $V(N_e)$ for $N_e < 1$, for $\delta_e = 0.3$ (solid), 0.5 (dash-dot), and 0.6 (dash), respectively. The parameters are $\alpha^2 = 0.8$ and $M_{Ad}^2 = 0.5$.

IV. LOCALIZED SOLUTIONS

We look for the quasistationary localized solutions of Eqs. (8)–(12). Accordingly we introduce the moving coordinate $\eta = \xi + \alpha\zeta - M_{Ad}\tau$, where α and M_{Ad}/α are direction cosine and the normalized (by v_{Ad}) speed of the new frame respect to the old one. Assuming a stationary structure in the new frame, we have $\partial_\xi = \partial_\eta$, $\partial_\zeta = \alpha \partial_\eta$, and $\partial_\tau = -M_{Ad} \partial_\eta$. After some algebra, we obtain the quadrature

$$\frac{1}{2} \left(\frac{dN_e}{d\eta} \right)^2 + V(N_e) = 0, \tag{15}$$

where the Sagdeev potential is

$$V(N_e) = N_e^2 \int_1^{N_e} \mathcal{F}(y) dy, \tag{16}$$

$$\mathcal{F}(y) = \frac{1 + M_{Ad}'^2}{y} - \frac{(1-\delta_e) i^{\sigma-1}}{1 - \delta_e y^{\sigma+1}} - \frac{M_{Ad}'^2 (1 - \delta_e y^{\sigma+1})}{(1-\delta_e) y^{\sigma+1}}, \tag{17}$$

and we have made use of the boundary conditions $\Phi \to 0$, $\Psi \to 0$, $\partial_\eta \Phi \to 0$, $\partial_\eta \Psi \to 0$, and $V(N_e) = \partial_{N_e} V(N_e) = 0$ for $N_j \to 1$. We have also defined the effective dust-Alfvén Mach number $M_{Ad}' = M_{Ad}/\alpha$. Eq. (15) describes a class of phaselocked (by the given boundary conditions) stationary finite amplitude DKAWs. One obtains localized solutions by applying in the final integration the condition $N_j = 1$ at $\eta \to \pm \infty$.

For arbitrary σ, it is difficult to get an explicit expression for $V(N_e)$. For many dusty plasmas, one has $T_e = T_i$, or $\sigma = 1$. In this case, we obtain

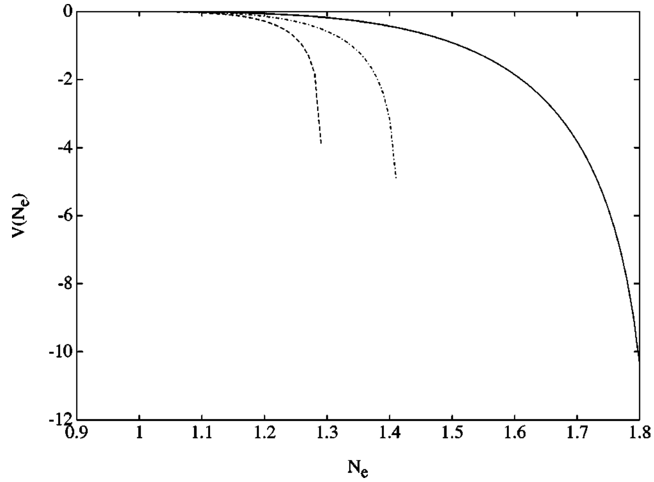

FIG. 2. $V(N_e)$ for $N_e > 1$ for the same parameters as in Fig. 1. The singularities (unmarked) occur at $N_e = \delta_e^{-1/2} =$ 1.83, 1.41, and 1.29 for $\delta_e =$ 0.3, 0.5, and 0.6, respectively.

$$V(N_e) = N_e^2\left(\frac{1-\delta_e}{\sqrt{\delta_e}}\tanh^{-1}\sqrt{\delta_e} - M_{Ad}'^2\frac{1+\delta_e}{1-\delta_e}\right) + (1+M_{Ad}'^2)N_e^2\ln N_e + \frac{M_{Ad}'^2}{(1-\delta_e)}(N_e+\delta_e N_e^3) - \frac{1-\delta_e}{\sqrt{\delta_e}}N_e^2\tanh^{-1}(\sqrt{\delta_e}N_e), \tag{18}$$

for the Sagdeev potential. Figures 1 and 2 give the Sagdeev potentials for $M_{Ad} = \sqrt{0.5}$, $\alpha = \sqrt{0.8}$, $Z_d = 10^4$, and $\delta_e =$ 0.3, 0.5, and 0.6. From Eq. (16), we have $V(N_e) \to \infty$ at $N_e = 1/\sqrt{\delta_e}$. Thus, in Fig. 2 the singularities occur at $N_e =$ 1.83, 1.41, and 1.29, for $\delta_e =$ 0.3, 0.5, and 0.6, respectively. In terms of the analogous integral of motion of a particle in a potential well, the particle is reflected at the infinite potential wall, so that the corresponding solution has a cusp at its peak [17]. Note that all the physical quantities are finite at the reflection point, although their derivatives may not be continuous there.

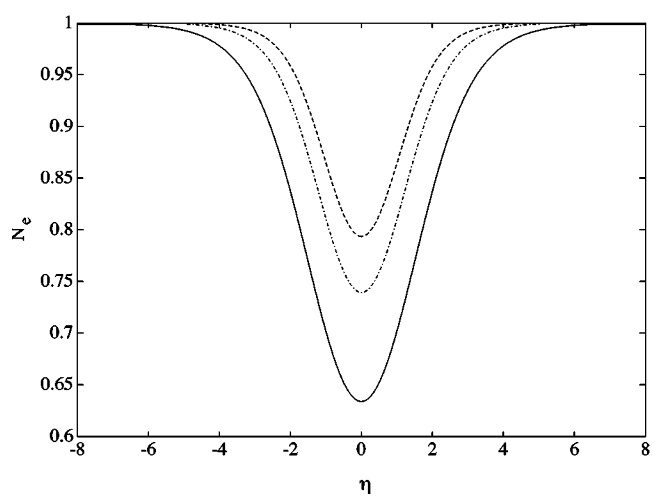

FIG. 3. Smooth solitons of electron density dips for the same parameters as in Fig. 1.

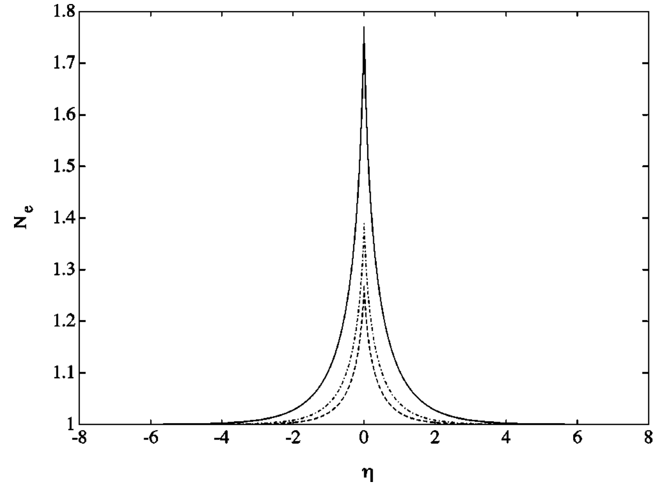

FIG. 4. Cusped solitons of electron density humps for the same parameters as in Fig. 1. The peak amplitudes (slightly off due to numerical inaccuracy near the singularities) correspond to the singularities of the Sagdeev potential.

V. DISCUSSION

The Sagdeev potentials (Figs. 1 and 2) show that localized smooth electron density dips and cusped density humps [17] coexist. Figures 3 and 4 give the profiles of these solitary wave solutions obtained by solving the ordinary differential equation (15) with $\sigma = 1$. The amplitude of the dips as well as the humps decreases with increasing δ_e. That is, the soliton amplitude increases with the dust number density. The halfwidth of the dips is in general larger than that of the cusped humps. A striking feature is that while the amplitude of the smooth solitons depends on all the plasma parameters as well as the propagation speed, as shown in Fig. 5, the amplitude of the cusped solitons depends only on the relative amount of dusts in the plasma. Such unique properties may make the DKAW solitons a useful tool in the diagnostics of dusts in magnetized plasmas. In real situations, however, the cusps (where the field gradients are large) are expected to be smoothed out somewhat by higher order nonlinearities or dissipative processes such as those arising from shear or tur-

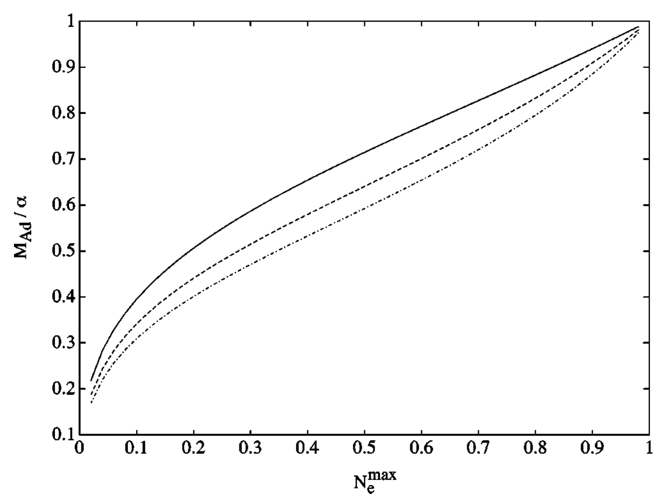

FIG. 5. $N_e^{\max}$ of the smooth solitons versus M_{Ad} relation for $\delta_e = 0.3$ (solid), 0.5 (dash), 0.6 (dash dot), and otherwise the same parameters as Fig. 1.

bulent viscosity. It is also possible that such solitons are destroyed completely by dissipative processes. In this case, their lifetime would be of the order of the shortest dust collision time, namely ν_{de}^{-1}, which is also related to the charging of the dusts.

Solitons with cusped profiles occur because of a balance of the dispersive and nonlinear effects close to the wave breaking point, and have been found in both fluids and plasmas [17–20]. But unlike the present case, usually the humps and dips do not coexist in the same parameter regime. It should be emphasized that although the Sagdeev potential for ordinary KAWs in an electron-ion plasma also predicts both density humps and dips [3,21], the KAW dips have at their maximum amplitudes exactly zero density. That is, all the plasma particles are expelled by the (infinite) local wave field. Such solutions are probably beyond the validity of the theory.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Project No. 19975043) and the Sonderforschungsbereich 191 Niedertemperatur Plasmen.

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